Applications of Exponential Equations (part 1):

1. Basic Exponential Model: $y = a(b)^x$ $\quad a = initial\ amount, \quad b = growth\ factor$

A Petri dish starts out with 10 bacteria cells. Each day the number of bacteria triples.

$a = 10\ cells, \quad b = 3\ (triples)$

a. Write the model. $y = 10(3)^x$

b. How many bacteria cells will there be in 5 days? $y = 10(3)^5 = 10 \cdot 243 = 2430$

c. How long until there are 100 cells? $100 = 10(3)^x \rightarrow 10 = 3^x$

write in log form: $x = log_3 10 = \frac{log10}{log3} = \frac{1}{0.477} = 2.096\ days$

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2. Half-Life: $Stuff = I\left(\frac{1}{2}\right)^{\frac{t}{\lambda}}$ $\quad I = initial\ amount,\ t = time,\ \lambda = half - life$

Note: When t = λ, then exponent = $\frac{\lambda}{\lambda} = 1$, so you will have $\frac{1}{2}$ or the initial amount.

The half-life of Potassium 42 is 12.36 hours. You have 10 grams of K-42 on the shelf.

a. Write the model: $Stuff = 10\left(\frac{1}{2}\right)^{\frac{t}{12.36}}$

b. How may grams of K-42 will be left in 2 days? $Stuff = 10\left(\frac{1}{2}\right)^{\frac{48}{12.36}} = 10\left(\frac{1}{2}\right)^{3.883} = 0.678\ grams$

c. How long until there is 2 grams left? $2 = 10\left(\frac{1}{2}\right)^{\frac{t}{12.36}} \rightarrow \quad 0.2 = (0.5)^{\frac{t}{12.36}}$

write in log form: $\frac{t}{12.36} = log_{0.5} 0.2 = \frac{\log 0.2}{\log 0.5} = 2.322 \rightarrow \quad t = 2.322 \cdot 12.36 = 28.70\ hours$

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3. Percent Increase: $P = I(1 + r)^t$ $\quad I$ = initial, r = rate (as a decimal), t = time

The population of Gophertown was 40,000 in 2000. The population increases by 12 percent each year.

a. Write the model. $P = 2000(1 + 0.12)^t = 2000(1.12)^t$

b. What will the population be in 2013? $P = 2000(1.12)^{13} = 8726\ people$

c. How long until the population triples? $6000 = 2000(1.12)^t \rightarrow \quad 3 = (1.12)^t$

write in log form: $t = log_{1.12} 3 = \frac{log3}{log1.12} = 9.694\ years$

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4. Simple Interest: $\$ = P(1 + r)^t$ P = Principal (initial investment), r = annual interest rate, t = time

You invest $1000 in a bank account that earns 2% interest each year.

a. Write the model. $\$ = 1000(1 + 0.02)^t = 1000(1.02)^t$

b. How much money will you have in 5 years? $\$ = 1000(1.02)^5 = 1104.08$

c. How long until your money double? $2000 = 1000(1.02)^t \rightarrow \quad 2 = (1.02)^t$

write in log form: $t = log_{1.02} 2 = \frac{log2}{log1.02} = 35\ years$